SIMULATION OF QUASILINEAR PARABOLIC EQUATIONS BY NETWORKS OF FIELD-EFFECT TRANSISTORS

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The possibility of using networks of field-effect transistors to simulate processes described by quasilinear parabolic equations is considered. Simulation results are compared with the data of a numerical calculation.

The use of R- and RC-networks to simulate processes described by the quasilinear parabolic equation

$$\frac{\partial}{\partial x}\left[A\frac{\partial T}{\partial x}\right]-B\frac{\partial T}{\partial t}=0,\ 0\leqslant x\leqslant l,\ t\geqslant t_0, \tag{1}$$

gives rise to a number of difficulties, not yet completely overcome, connected with the need to realize the prescribed functional relationships A = A(T), B = B(T) [1, 2]. For certain commonly occurring forms of A(T) and B(T) this problem can be solved by using RC-networks in which the nonlinear active resistances are various types of field-effect transistors (FET's).

Consider for simplicity the case when the coefficient B does not depend on T (i.e., $B = B_0$ = const). [The case when B depends on T is readily reduced to the constant-B case by means of the substitution $T^* = \int B(T)dT$.] By dividing the segment $[0, l]$ into n equal parts and replacing the spatial derivatives by finite-difference relationships, we arrive at a set of ordinary differential equations of the following type [3]:

$$\frac{n^2}{l^2}\left[\int\limits_{T_{i+1}}^{T_i} A(T)\,dT-\int\limits_{T_i}^{T_{i-1}} A(T)\,dT\right]=-B_0\frac{dT_i}{dt},\ i=1,2,\ldots,n-1. \tag{2}$$

We introduce generalized dimensionless variables $\bar{A}$, $\bar{T}$, $\bar{l}$, $\bar{t}$ by means of the relationships

$$A(T)=A_0\bar{A}(T),\ T=T_0\bar{T},\quad l=l_0\bar{l},\quad t=t_0\bar{t}. \tag{3}$$

As a result we obtain

$$\left[\int\limits_{\bar{T}_{i+1}}^{\bar{T}_i}\bar{A}(T)\,d\bar{T}-\int\limits_{\bar{T}_i}^{\bar{T}_{i-1}}\bar{A}(T)\,d\bar{T}\right]=-\frac{B_0 l_0^2\bar{l}^2}{A_0 n^2 t_0}\cdot\frac{d\bar{T}}{d\bar{t}},\ i=1,2,\ldots,n-1. \tag{4}$$

To simulate (4) we utilize the network shown in Fig. 1. The following condition holds at each node of the network:

$$I_{i+1}-I_i=-C_0\frac{dV_i}{d\tau}. \tag{5}$$

We transform to dimensionless variables $\bar{I}_i$, $\bar{V}_i$, $\bar{\tau}$ by means of the relationships

$$I_i=I_0\bar{I}_i,\ V_i=V_0\bar{V}_i,\ \tau=\tau_0\bar{\tau}, \tag{6}$$

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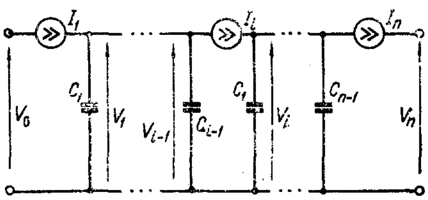

Fig. 1

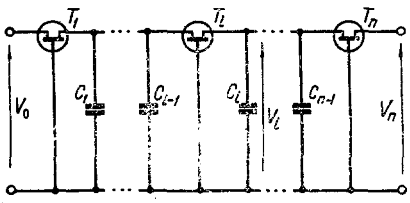

Fig. 2

Fig. 1. Electrical model of medium with nonlinear transport parameters.

Fig. 2. Network using FET's for simulation of processes described by Eq. (1).

$$\bar{I}_{i+1} - \bar{I}_i = -\frac{C_0 V_0}{I_0 \tau_0} \cdot \frac{d\bar{V}_i}{d\tau}. \tag{7}$$

On comparing (4) and (7) we find

$$\frac{l^2}{n^2} \cdot \frac{B_0}{A_0 t_0} = \frac{C_0 V_0}{I_0 \tau_0}, \tag{8}$$

$$I_i = \frac{I_0}{V_0} \int_{V_i}^{V_{i-1}} \bar{A}\left(\frac{T_0}{V_0} V\right) dV. \tag{9}$$

Expression (9) coincides with the expression describing the steep part of the volt–ampere characteristic of the field-effect transistor, for which the dependence of the running resistance of the channel R(V) on the channel-gate voltage V is given by

$$R(V) = \frac{V_0}{I_0 L_C \bar{A}\left(\frac{T_0}{V_0} V\right)}, \tag{10}$$

where L_C is the channel length, and voltages V_{i-1} and V_i play, respectively, the roles of the source-gate and drain-gate voltage.

For transistors with a controlling p–n junction and for insulated-gate FET's the functions R(V) have the following form [4]:

$$R(V) = \frac{1}{\sigma z a}\left[\frac{1}{1 - \sqrt{\frac{V}{V_p}}}\right], \tag{11}$$

$$R(V) = \frac{L_C}{\mu C_t (V - V_p)}. \tag{12}$$

Replacing in Fig. 1 current sources of FET's of the appropriate type, we obtain a network for the simulation of processes for which the functional dependence A(T) is approximated by expressions (10), (11) or (10), (12).

By way of example, consider the temperature distribution in a single-layer diatomic wall of thickness 0.2 m under the following boundary conditions:

$$T(x, 0) = 0°C, \; T(0, t) = T_{in} = 600°C, \; \left.\frac{\partial T(x, t)}{\partial x}\right|_{x=l} = 0. \tag{13}$$

The temperature dependence of the thermal conductivity is given by the expression $\lambda = 0.116 + 0.00018T$ W/m·deg; the specific heat $c = 1.05 \cdot 10^3$ J/kg·deg; and the density $\gamma = 560$ kg/m^3 [5].

The simulation was carried out using a network (Fig. 2) comprising 10 KP301 metal-dielectric FET's selected so that the spread in the slopes and pinch-off voltages did not exceed 4%. The effect of the stray capacitance of the transistors on the duration of the transient process was eliminated by choosing C_0 equal to 220 pF, i.e., much greater than the transistor interelectrode capacitances. In Fig. 3 the results of the measurements (the circles) are compared with the results of a numerical solution (solid curves). The deviation between the results of simulation and numerical calculation lies within the limits of measurement error and

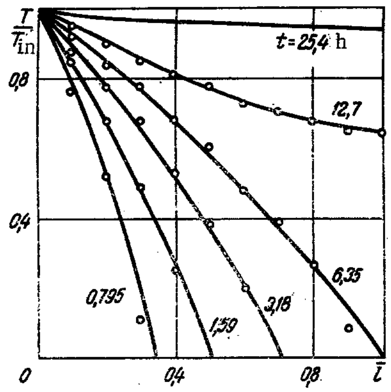

Fig. 3. Temperature distribution in single-layer wall.

does not exceed 6%. By an analogous approach the method described above can be used for the simulation of a wide range of problems in heat and mass transport.

NOTATION

t, time; T, temperature; T_{in}, initial temperature; x, coordinate; L_C, z, a, σ, length, width, thickness, and conductivity of channel, respectively; μ, mobility of charge carriers; C_t, total gate-channel capacitance; V_p, pinch-off voltage for insulated gate (controlling p–n junction) transistor; I, current; V, voltage; C_0, capacitance; λ, thermal conductivity; c, specific heat; γ, density; $\bar{I}$, $\bar{V}$, $\bar{\tau}$, $\bar{A}$, $\bar{T}$, $\bar{l}$, $\bar{t}$, dimensionless variables; I_0, V_0, τ_0, A_0, T_0, l_0, t_0, constant coefficients.

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